Right Angled Triangles and the Pythagorean Theorem

One of the oldest and most widely used mathematical formulas is the Pythagorean Theorem. It concerns the relationships between the lengths of the sides of a right-angled triangle. It bears the name of the Greek mathematician, Pythagoras, who discovered this formula around 500 BC.

The Basics

The theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. To understand the theorem, we'll first 'unpack' what this sentence means.

A 'right-angled triangle' is a triangle where one of the angles is equal to exactly 90º.

The 'hypotenuse' is the side of a right-angled triangle which is opposite the 90º angle. Only right-angled triangles have a hypotenuse.

Here the term 'the square' means a number multiplied by itself such as 5x5 or 25. In algebra the square is written as x^2.

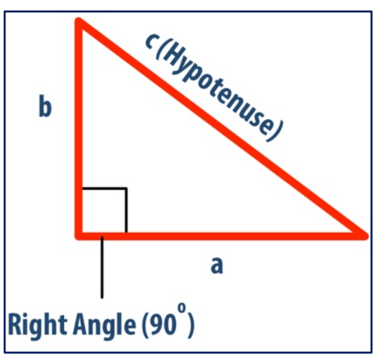

Figure 1: A Right-Angled Triangle

With these definitions and the triangle in Figure 1, we can state the Pythagorean Theorem as:

$a^2 + b^2 = c^2$ where c is the hypotenuse.

Equation 1: The Basic Trigonometric Functions

If you take the square root of both sides this can also be written as:

$c = \sqrt{a^2 + b^2}$

Equation 2: Another Version of the Pythagorean Theorem

This second equation states that the length of the hypotenuse is equal to the square root of the sum of the squares of the other two sides.

The square root of a number is another number which when multiplied by itself equals the number. So, for example, 5 is the square root of 25 and 25 is the square of 5. 6 is the square root of 36 and 36 is the square of 6.

TIP: When preparing for a course in structures make sure you have a calculator that can calculate squares and square roots.

Pythagorean Triples

Some right-angled triangles have sides whose lengths are all integers (in other words they have no decimals or fractions). These triangles are called Pythagorean Triples. Some of the basic ones were even known to the ancient Egyptians as early as 3000 BC. They used the 3-4-5 triple in knotted ropes to lay out right angled triangles when surveying the land. The table below shows the most common triples. In each case the largest number is always the hypotenuse.

3-4-5
5-12-13
7-24-25
8-15-17

Table 1: Common Trigonometric Values (There are many more)

You can check any of these using the theorem:

$5^2 + 12^2 = 13^2$
or
$25 + 144 = 169$ (which is true)

TIP: Exam questions often use Pythagorean Triples so if you remember the common ones (like 3-4-5 and 5-12-13) you can sometimes save yourself a lot of calculation time.

Example 1: Finding the Value of the Hypotenuse with the Pythagorean Theorem

We can use the Pythagorean Theorem to calculate the hypotenuse of a right-angled triangle.

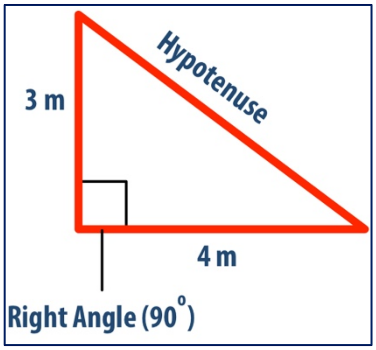

Figure 2: Calculating the Hypotenuse

In this simple example, we are given two sides of a right-angled triangle and we need to find the length of the hypotenuse.

From Equation 2 (above) we know that:

$c = \sqrt{a^2 + b^2}$

or

$\text{Hypotenuse} = \sqrt{3^2 + 4^2}$

or

$\text{Hypotenuse} = \sqrt{9 + 16}$

or

$\text{Hypotenuse} = \sqrt{25}$

or

$\text{Hypotenuse} = 5$

So we know that the length of the hypotenuse is 5 m.

We often use the Pythagorean Theorem in conjunction with Trigonometric Functions (see the Open Educational Resource called *The Basics of Trigonometry*), to calculate the direction and magnitude of forces in structures.

Additional Resources

The website http://www.radford.edu/~wacase/Math%20135%20Pythagorean%20Theorem.pdf provides an in-depth review of the theorem including its proof and a number of examples.

If you want a more entertaining view of the theorem, you may also want to check out the Pythagorean Theorem Song by Colin Dodds at https://www.youtube.com/watch?v=l8-bnZh8Zuc